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Net voltage caused by correlated symmetric noises

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A Gaussian white noise model and a symmetric dichotomous noise model for superconducting (Josephson) junctions are studied. We show that correlated symmetric noises can produce a net voltage, which stems from a symmetric breaking of the system induced by the correlation between additive and multiplicative noises. It is found that there is a negative net voltage, exhibiting a peak with increasing noise strength. The results provide a theoretical foundation for reducing the net voltage caused by the correlated symmetric noises. [S1063-651X(98)15106-1]

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I. INTRODUCTION

Superconducting tunnel junctions are interesting, not only because of their broad range of applications, but also because they are nice model systems for studying a number of important types of nonlinear phenomena, such as phase locking, bifurcations, chaos, solitonic excitations, and pattern formation [1]. Josephson showed that, in addition to the usual single-particle tnnneling, electron pairs can tunnel through a narrow insulating material between two superconductors [2]. Such a superconducting (Josephson) junction consists of a junction shunted by a resistance R, and driven by a current $I(t)$. The pair current across the junction is given by $J_c = J_0 \sin\phi$, where J_0 is the critical current and ϕ is the phase difference of the superconducting order parameters across the junction. The evolution of the phase difference can be described by the equation

$$\frac{\hbar}{2eR}\dot{\phi} + J_0 \sin\phi = I(t), \tag{1}$$

where the phase difference is related to the voltage by $V(t) = (\hbar/2e)\dot{\phi}$, and $I(t)$ is a driving current [3]. Here we will be interested in totally unbiased driving $\langle I(t)\rangle = 0$, where $\langle\ \rangle$ stands for the time average.

Millonas and Chialvo [4] showed that an asymmetry in the spectral properties of the noise (we call it the asymmetric noise) can result in a fluctuation-induced net voltage in the superconducting tunnel junction,

$$\langle V(t)\rangle = \frac{\hbar}{2e}\langle\dot{\phi}\rangle. \tag{2}$$

At the same time, they suggested that any noise with a uniform distribution of phases, such as Gaussian noise, will be symmetric, and will not give rise to a net voltage. However, this conclusion was drawn by considering the driving current as an additive noise, which does not appear universal. In this paper, we study the net voltage caused by environmental perturbations, together with the fluctuation of the driving current. The environmental perturbations can be described by the multiplicative noise in the Langevin equation [5–7]. Taking into account both the additive and multiplicative noises, we find that in some circumstances the symmetric noises can produce a net voltage.

II. GAUSSIAN WHITE NOISE MODEL

Environmental perturbations, such as the perturbation of electromagnetic fields, the external vibration, and the change of the external temperature, will give rise to a fluctuation of the critical current in the Josephson junction. We describe this fluctuation by a stochastic external parameter $J_0 + \sigma\xi_0(t)$, in which $\xi_0(t)$ is the stochastic force of the Gaussian white noise and σ is a positive constant. On the other hand, the stochastic driving current is taken to be a thermal Gaussian white noise $I(t) = \eta_0(t)$. Then Eq. (1) becomes

$$\frac{\hbar}{2eR}\dot{\phi} + [J_0 + \sigma\xi_0(t)]\sin\phi = \eta_0(t), \tag{3}$$

where $\langle\xi_0(t)\rangle_f = \langle\eta_0(t)\rangle_f = 0$, $\langle\xi_0(t)\xi_0(t')\rangle_f = 2D'\delta(t-t')$, and $\langle\eta_0(t)\eta_0(t')\rangle_f = 2D\delta(t-t')$, with $\langle\ \rangle_f$ denoting the average over stochastic force. The multiplicative noise $\xi_0(t)$ and the additive noise $\eta_0(t)$ in the Josephson junctions come from external environmental perturbation and thermal perturbation, respectively. However, they are not independent, but corrected to each other. For example, the environmental perturbation can also lead to a change of the thermal vibration of molecules in the Josephson junction. Such fluctuation effects of the molecular vibrations will affect the thermal additive noise. Thus we consider the correlation between the thermal noise $\xi_0(t)$ and the external noise $\eta_0(t)$, and assume that their correlation function is taken to be a simple relation, $\langle\eta_0(t)\xi_0(t)\rangle_f = 2\lambda\sqrt{DD'}\,\delta(t-t')$, with $0 \leqslant \lambda \leqslant 1$ indicating the strength of the correlation. Equation (3) can be rewritten as

$$\dot{\phi} = -\omega_0 \sin\phi - \xi(t)\sin\phi + \eta(t), \tag{4}$$

where $\omega_0 = 2eRJ_0/\hbar$, $\xi(t) = (2e\sigma R/\hbar)\xi_0(t)$, and $\eta(t) = (2eR/\hbar)\eta_0(t)$. The Stratonovich interpretation of the stochastic differential equation (4) yields the Fokker-Planck equation [8,9]

$$\partial_t P(\phi,t) = -\partial_\phi A(\phi) P(\phi,t) + \partial_\phi^2 B(\phi) P(\phi,t), \quad (5)$$

where $A(\phi) = -\omega_0 \sin\phi - \tilde{D}' \cos\phi(-\sin\phi + \lambda\sqrt{\tilde{D}/\tilde{D}'})$, and $B(\phi) = \tilde{D}(1-\lambda^2) + \tilde{D}'(-\sin\phi + \lambda\sqrt{\tilde{D}/\tilde{D}'})^2$ with $\tilde{D} = (2eR/\hbar)^2 D$ and $\tilde{D}' = (2e\sigma R/\hbar)^2 D'$. Under periodic boundary conditions, we obtain the stationary solution of Eq. (5) [10,11]

$$P_s(\phi) = N \frac{e^{\Psi(\phi)}}{B(\phi)} \oint d\phi' e^{-\Psi(\phi') - \mu\theta(\phi-\phi')}. \quad (6)$$

Here

$$\Psi(\phi) = \int_0^\phi [A(\phi')/B(\phi')] d\phi',$$

$$\mu = \omega_0 \int_0^{2\pi} \frac{\sin\phi}{\tilde{D}(1-\lambda^2) + \tilde{D}'(-\sin\phi + \lambda\sqrt{\tilde{D}/\tilde{D}'})^2} d\phi,$$

$\theta(\phi-\phi')$ is the Heaviside step function, and N is a normalized constant.

The net voltage is given by

$$\langle V(t)\rangle = \langle\langle V(\phi,t)\rangle_\phi\rangle_f = \frac{\hbar}{2e}\langle\langle \dot{\phi}\rangle_\phi\rangle_f$$
$$= \frac{\hbar}{2e}\langle\langle -\omega_0 \sin\phi - \xi(t)\sin\phi\rangle_f\rangle_\phi, \quad (7)$$

where $\langle\ \rangle_\phi$ stands for the average over ϕ. According to the Novikov theorem [12], we have (see the Appendix)

$$\langle \xi(t)\sin\phi\rangle_f = \tilde{D}'(-\sin\phi + \lambda\sqrt{\tilde{D}/\tilde{D}'})\cos\phi. \quad (8)$$

From Eqs. (7) and (8), we obtain

$$\langle V\rangle_s = \lim_{t\to\infty}\frac{1}{t}\int_0^t \langle\langle V(\phi,\tau)\rangle_\phi\rangle_f d\tau = \frac{\hbar}{2e}\langle A(\phi)\rangle_\phi$$
$$= \frac{\hbar}{2e}\int_0^{2\pi} P_s(\phi) A(\phi) d\phi$$
$$= \frac{N\hbar}{2e}\oint d\phi \frac{A(\phi)e^{\Psi(\phi)}}{B(\phi)} \oint d\phi' e^{-\Psi(\phi') - \mu\theta(\phi-\phi')}. \quad (9)$$

It can be seen from Eq. (9) that if the additive and multiplicative noises are uncorrelated ($\lambda = 0$), the net voltage will be zero; the net voltage always exists provided $\lambda \neq 0$.

We wish to give some explanations of the origin of the net voltage. Consider a solution $\phi(t)$ of Eq. (4) for a given realization of the noises. Then $-\phi(t)$ is also a solution of Eq. (4), with $\eta(t)$ replaced by $-\eta(t)$. If $\eta(t)$ and $\xi(t)$ were uncorrelated, this solution $-\phi(t)$ would have the same probability as $\phi(t)$, and there would be no symmetric breaking. However, in the presence of the correlation, the probability, even though it is Gaussian, does not have this symmetry. The result obtained here is somewhat similar to that obtained by Milonas and Chialvo [4], but the present asymmetry is induced by the correlation between the additive and multiplicative noises.

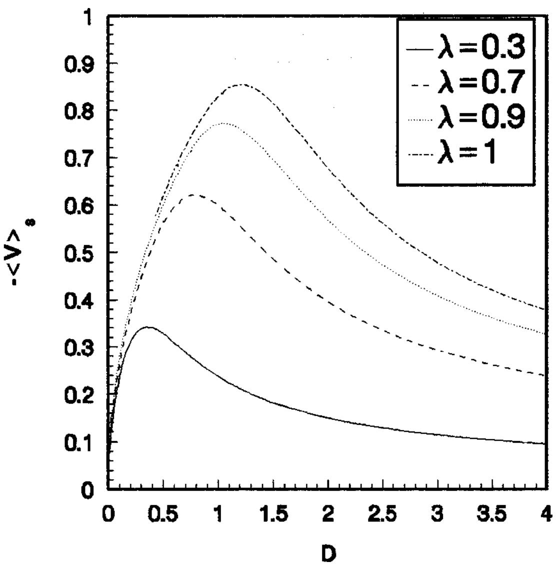

FIG. 1. The mean negative voltage vs D for different values of λ in the dimensionless form. $D' = 0.3$ and $\omega_0 = 1$ are fixed and $\lambda = 0.3$, 0.7, 0.9, and 1.

In what follows we calculate $\langle V\rangle_s$ from Eq. (9) by using dimensionless parameters and setting $\hbar/2e = J_0 = \sigma = R = 1$. In Fig. 1, we plot the negative net voltage versus the additive noise strength D for different values of λ. Here we set $D' = 0.3$, $\omega_0 = 1$, and $\lambda = 0.3$, 0.7, 0.9, and 1, respectively. When $\lambda = 1$, the stationary probability density $P_s(\phi)$ [see Eq. (6)] will be divergent at the points $\phi = \sin^{-1}\sqrt{D/D'} + 2n\pi$ ($n = 0, \pm 1, \pm 2, \pm 3, \ldots$). For $D < D'$, $\langle V\rangle_s$ cannot be determined from Eq. (9); hence we only plot the curve in the case of $D > D'$. Several distinct featues can be seen from Fig. 1. First, the net voltage is always negative. Second, there is a peak in the $\langle V\rangle_s$ vs D curve. Third, with the increase of λ, the net voltage increases, and the peak value moves toward the right. In Fig. 2, we represent the net voltage as a function of D for different multiplicative noise strength D'. It is found that as the value of D' increases, the crest value descends, and the curve becomes more and more smooth.

III. A DICHOTOMOUS NOISE MODEL

If $\xi_0(t)$ in Eq. (3) is replaced by a symmetric dichotomous noise [we now set the driving current $I(t) = a\xi_0(t)$], we have

$$\dot{\phi} = -\omega_0 \sin\phi - \xi(t)\sin\phi + \tilde{a}\xi(t), \quad (10)$$

which has the same form as Eq. (6) of Ref. [4]. Here $\xi(t) = (2e\sigma R/\hbar)\xi_0(t)$, $\omega_0 = 2eRJ_0/\hbar$, and $\tilde{a} = a/\sigma$. The statistical properties of $\xi_0(t)$ are now given by

$$\langle \xi_0(t)\rangle_f = 0, \quad \langle \xi_0(t)\xi_0(t')\rangle_f = \frac{D}{\tau} e^{-|t-t'|/\tau} = E^2 e^{-\lambda'|t-t'|}. \quad (11)$$

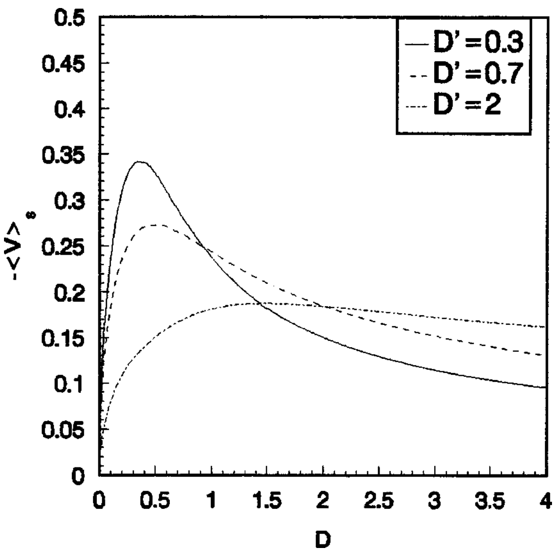

FIG. 2. The mean negative voltage vs D for different values of D' in the dimensionless form. $\lambda=0.3$, $\omega_0=1$, and $D'=0.3$, 0.7, and 2.

Here $\xi_0(t)$ has two values $\pm E$ ($E>0$), and $1/2\tau=\lambda'/2$ is the transition probability of $\xi_0(t)$ from E to $-E$, or vice versa. From Eq. (10), we can obtain the stochastic Liouville equation [13]

$$\partial_t\rho(\phi,t)=-\partial_\phi[-\omega_0\sin\phi-\sin\phi\xi(t)+\tilde{a}\xi(t)]\rho(\phi,t). \tag{12}$$

Since the probability density is given by $P'(\phi,t)=\langle\rho(\phi,t)\rangle_f$, it follows from Eq. (12) that

$$\partial_t P'(\phi,t)=\partial_\phi\omega_0\sin\phi P'(\phi,t)+\partial_\phi(\sin\phi-\tilde{a})P_1'(\phi,t), \tag{13}$$

where $P_1'(\phi,t)=\langle\xi(t)\rho(\phi,t)\rangle_f$. From the Shapino-Loginov differential formula [14], we obtain

$$\partial_t P_1'(\phi,t)=\omega_0\partial_\phi\sin\phi P_1'(\phi,t)+\tilde{E}^2\partial_\phi(\sin\phi-\tilde{a})P'(\phi,t)-\lambda' P_1'(\phi,t), \tag{14}$$

with $\tilde{E}=(2e\sigma R/\hbar)E$.

The formal solution of Eq. (14) is

$$P_1'(\phi,t)=\int_0^t e^{-\hat{A}(t-\tau)}[\tilde{E}^2\partial_\phi(\sin\phi-\tilde{a})P'(\phi,\tau)]d\tau, \tag{15}$$

where $\hat{A}=\lambda'-\omega_0\partial_\phi\sin\phi$. Substituting Eq. (15) into Eq. (13), we obtain the probability density equation

$$\partial_t P'(\phi,t)=\partial_\phi\omega_0\sin\phi P'(\phi,t)+\partial_\phi(\sin\phi-\tilde{a})\times\int_0^t e^{-\hat{A}(t-\tau)}\tilde{E}^2\partial_\phi(\sin\phi-\tilde{a})P'(\phi,\tau)d\tau. \tag{16}$$

In the case of $\lambda'\gg\omega_0$, the integral on the right-hand side of Eq. (16) stems mainly from the values near $t=\tau$, so we approximately take $P'(x,\tau)\simeq P'(x,t)$. Then Eq. (16) becomes

$$\partial_t P'(\phi,t)=\partial_\phi\omega_0\sin\phi P'(\phi,t)+\partial_\phi(\sin\phi-\tilde{a})\times\hat{A}^{-1}\tilde{E}^2\partial_\phi(\sin\phi-\tilde{a})P'(\phi,t). \tag{17}$$

Taking into account the condition $\lambda'\gg\omega_0$, we obtain the approximate Fokker-Planck equation

$$\partial_t P'(\phi,t)=-\partial_\phi A_1(\phi)P'(\phi,t)+\partial_\phi^2 B_1(\phi)P'(\phi,t), \tag{18}$$

where $A_1(\phi)=-\omega_0\sin\phi-(\tilde{E}^2/\lambda')\cos\phi(\tilde{a}-\sin\phi)$, and $B_1(\phi)=(\tilde{E}^2/\lambda')(\tilde{a}-\sin\phi)^2$. With the periodic boundary conditions, the stationary solution of Eq. (18) is given by

$$P_s'(\phi)=N_1\frac{e^{\Psi_1(\phi)}}{B_1(\phi)}\oint d\phi' e^{-\Psi_1(\phi')-\mu_1\theta(\phi-\phi')}, \tag{19}$$

where $\Psi_1(\phi)=\int_0^\phi[A_1(\phi')/B_1(\phi')]d\phi'$, $\mu_1=(\lambda'\omega_0/\tilde{E}^2)\int_0^{2\pi}[\sin\phi d\phi/(\tilde{a}-\sin\phi)^2]$, and N_1 is the normalized constant. From Eq. (19), we find that when $|\tilde{a}|\leq 1$, the stationary state of the system will be divergent at the points $\phi=\sin^{-1}\tilde{a}+2\pi n$ ($n=0,\pm1,\pm2,\pm3,\dots$). As a result, we only study the case of $|\tilde{a}|>1$. In this case, the net voltage is given by

$$\langle V(t)\rangle=\langle\langle V(\phi,t)\rangle_\phi\rangle_f=\frac{\hbar}{2e}\langle\langle-\omega_0\sin\phi-\xi(t)\sin\phi\rangle_f\rangle_\phi. \tag{20}$$

Using the Shapino-Loginov differential formula [14], we obtain

$$\langle\sin\phi\xi(t)\rangle_f=\sum_{k=1}^{\infty}\frac{(-1)^k\tilde{E}^2\omega_0^{k-1}}{(2\lambda')^k}\cos 2^{k-1}\phi(1-\sin 2^{k-1}\phi), \tag{21}$$

so that

$$\begin{aligned}\langle V\rangle_s&=\frac{\hbar}{2e}\Bigg(-\omega_0\langle\sin\phi\rangle_\phi+\sum_{k=1}^{\infty}\frac{(-1)^k\tilde{E}^2\omega_0^{k-1}}{(2\lambda')^k}\times\langle\cos 2^{k-1}\phi(1-\sin 2^{k-1}\phi)\rangle_\phi\Bigg)\\&=-\frac{\hbar\omega_0}{2e}\int_0^{2\pi}P_s'(\phi)\sin\phi d\phi+\frac{\hbar}{2e}\sum_{k=1}^{\infty}\frac{(-1)^k\tilde{E}^2\omega_0^{k-1}}{(2\lambda')^k}\times\int_0^{2\pi}P_s'(\phi)\cos 2^{k-1}\phi(1-\sin 2^{k-1}\phi)d\phi.\end{aligned} \tag{22}$$

In Eq. (10), the multiplicative and additive noises are taken to be the same Gaussian white noise for ease of calculation. If they are taken as different Gaussian white noises, Eq. (10) is replaced by

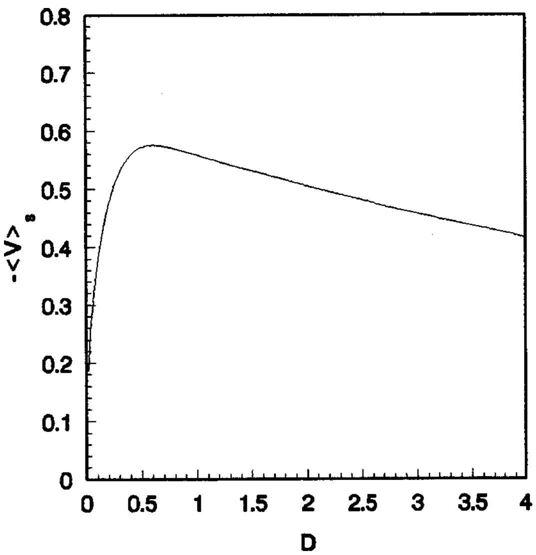

FIG. 3. The mean negative voltage vs D in the dimensionless form. $\lambda'=100$, $\omega_0=1$, and $a=2$.

$$\dot{\phi}=-\omega_0\sin\phi-\xi(t)\sin\phi+\tilde{a}\,\eta(t), \tag{23}$$

where $\xi(t)$ and $\eta(t)$ have the values $\pm\tilde{E}$ and $\pm\tilde{E}'$ ($\tilde{E}\neq\tilde{E}'$, and $\tilde{E},\tilde{E}'>0$), respectively. We set their statistical properties to be $\langle\xi(t)\rangle_f=\langle\eta(t)\rangle_f=0$, $\langle\xi(t)\xi(t')\rangle_f=\tilde{E}^2e^{-\lambda'|t-t'|}$, $\langle\eta(t)\eta(t')\rangle_f=\tilde{E}'^2e^{-\lambda'|t-t'|}$, and $\langle\xi(t)\eta(t')\rangle_f=\alpha\sqrt{\tilde{E}\tilde{E}'}e^{-\lambda'|t-t'|}$ $(0\leqslant\alpha\leqslant1)$. The transition probability for $\xi(t)$ from $\tilde{E}$ to $-\tilde{E}$, or vice versa, is $\lambda'/2$; the corresponding transition probability for $\eta(t)$ is also $\lambda'/2$. It is clear that Eq. (10) is a special form of Eq. (23) in the case of $\alpha=1$ and $\tilde{E}=\tilde{E}'$. Further study shows that in the symmetric dichotomous noise case, the net voltage appears only when the multiplicative noise and the additive noise have a correlation.

Figure 3 shows the calculated result of Eq. (10), in which the net voltage is negative and exhibits a peak with increasing D. In the calculation we only take the preceding four terms in the expansion of Eq. (22).

IV. DISCUSSION

In the two models above, we find that the net voltage caused by the correlated symmetric noises is always negative. From the calculated results, one may manipulate the noises to reduce the net voltage to the lowest degree. In the case of given environmental perturbation, for the first model, we may appropriately adjust temperature to make the net voltage depart from the peak value by taking into account that in Eq. (3) the internal thermal noise strength D is proportional to the temperature. For the second model, we may change the intensity of the driving current so as to make the net voltage leave the peak value. For given temperature and driving current, we should adopt measures to reduce and avoid the environmental perturbation in order to make the net voltage the lowest.

It was reported [4] that symmetric noise cannot induce a net voltage in the Josephson junction. By the study of the above two models, we find that if in the system there is a correlation between additive and multiplicative noises, the symmetric noise can also produce a net voltage in the Josephson junction. The energy in response to the net voltage arises from the noise's energy, which is determined by the correlation between the additive and multiplicative noises. If the noises in Eq. (3) are *O-U* noises or symmetric shot noises, the same result will be obtained, provided that there is a correlation between the additive and multiplicative noises. The key factors of the symmetric noise-induced net voltage are (a) the additive and multiplicative noises must exist simultaneously, and (b) there exists a correlation between the additive and multiplicative noises. Such a correlation breaks the symmetry of the system, and makes the probabilities of the fluctuations on the two sides of the potential barrier different, so that a net voltage arises.

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APPENDIX

Equation (4) can be transformed into the following form [8,9]:

$$\dot{\phi}=-\omega_0\sin\phi+(-\sin\phi+\lambda\sqrt{\tilde{D}/\tilde{D}'})\xi(t)+\eta'(t), \tag{A1}$$

where $\eta'(t)=\eta(t)-\lambda\sqrt{\tilde{D}/\tilde{D}'}\xi(t)$. The statistical properties of $\eta'(t)$ are $\langle\eta'(t)\rangle_f=0$, and $\langle\eta'(t)\eta'(t')\rangle_f=2\tilde{D}(1-\lambda^2)\delta(t-t')$. Now the noises $\xi(t)$ and $\eta'(t)$ are no longer correlated.

From Eq. (A1) and the Novikov theorem [12], we can obtain

$$\langle\xi(t)\sin\phi\rangle_f=\tilde{D}'(-\sin\phi+\lambda\sqrt{\tilde{D}/\tilde{D}'})\cos\phi. \tag{A2}$$

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